Code No. 1821

FACULTY OF SCIENCE

M.Sc. (Mathematics) II – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

ADVANCED ALGEBRA

PAPER – I

Time: 3 Hours] [Max. Marks:70

Note: Answer all questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. State and prove Eiseustein criterion.
2. Prove that Q in a prime field.
3. If E is the splitting field of $x^4 - 2 \in Q[x]$ over Q then complete $G(E/Q)$
4. Compute $\phi_4(x)$
5. Show that the polinomial $x^7-10x^5+15x+5$ is not solvable by radical over Q

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) i) Let E be an algebraic extension of a field F and let $\sigma: F \to L$ be an embedding of F into algebraically closed field L. Then prove that σ can be extended to an embedding $n : E \to L.$

(OR)

b) Let p(x) be an irreducible polynomial in F[x] and let u be a root of p(x) in an extension E of F. then prove that $\{1, u, u^2, \ldots\ldots, u^{n-1}\}$ forms a basis of F(u) over F where n is the degree of p(x).

7. a) If E is a finite separable extention of a field F, then prove that E is a simple extension of F.

(OR)

b) i) Prove that the multiplicative group of nonzero elements of finite field is cyclic.
ii) If $[E : F] < \infty$, $| F | < \infty$ then prove that E is a simple extension of F

8. a) State and prove the fundamental theorem of Galoes theory.

(OR)

b) State and prove the fundamental theorem of algebra.

9. a) State and prove the Lamma related to a special case of Hilbert's problem 90.

(OR)

b) Prove that $f(x) \in F[x]$ is solvable by radicals over F if and only if its splitting field E over F has solvable Galos group G(E/F).

10. a) Let ω be a primitive n^{th} root of unity in C. Then prove that $Q(\omega)$ is the splitting field of $\phi_n(x)$and also of $x^n-1 \in Q[x]$. Also prove that $[Q(\omega) : Q] = \varphi(n) = |G(Q(\omega)/Q)|$ and $G(Q(\omega)/Q) \simeq \left(\frac{z}{(n)}\right)^*$

(OR)

b) Let E be a finite separable extension of a field F. Then prove that the following are equivalent :
(i) E is a normal extension of F.
(ii) F is the fixed field of G (E/F).
(iii) $[E : F] = | G(E/F) |$.

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Code No. 1822

FACULTY OF SCIENCE

M.Sc. (Mathematics) II – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

ADVANCED REAL ANALYSIS

PAPER – II

Time: 3 Hours] [Max. Marks: 70

Note: Answer all questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. Define Lebesgue measurable set. Suppose E is a subset of R such that $m^*(E) = 0$ then prove that E is measurable.
2. Define Lébesgue integral of a bounded measurable function defined on a measurable set E with $M(E) < \propto$. Suppose f, g are bounded measurable functions defined on measurable set E of finite measure prove that $\int_E (f+g) = \int_E f + \int_E g$
3. Suppose p,n,t denote positive, negative and total variation sum of f on[a,b] prove that (i) p-n=f(b)-f(0) (ii) p+n=t.
4. If x is a finite dimensional vector space and if x_1 is a vector space in x then prove that there exists a projection P in x such that $R(P)=x_1$
5. State and prove Holder's inesuality.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Prove the following
   (i) If E1, E2 are measurable than $E_1 \cup E_2$ is measurable.
   (ii) Suppose A is any subset of R and $E_1, E_2 \ldots E_n$ is any finite collection of pairwise disjoint measurable sets in R then $m\left(A \cap \left(\cup_{j=1}^{n} E_j\right)\right) = \sum_{j=1}^{n} m^*(A \cap E_j)$

   (OR)

   b) Show that there exists a non measurable subset of R.
7. a) State and prove bounded convergence theorem.

   (OR)

   b) Suppose f is a non-negative measurable function which is integrable over a measurable set E then prove that given any $\in>0$ there exists a $\delta>0$ such that for all subsets $A \subseteq E$ with $m(A) < \delta$ we have $\int_A f < E$.
8. a) Prove that a real valued function f defined on [a, b] is of bounded variation on [a, b] if and only if f is the difference of two monotonically increasing functions.

   (OR)

   b) Suppose F is a bounded measurable function on [a,b] Let $F(x)=F(a)+\int_a^x f(t)dt \ \forall\ x \in [a,b]$ then prove that $F^1(x)=f(x)$ are on [a,b].
9. a) State and prove inverse function theorem.

   (OR)

   b) (i) Prove that an linear operator A on R^n is invertible if and only if det $[A] \neq 0$

   (ii) Suppose f is defined in an open set $E \subset R^2$ and $D_1 f$ and $D_{21} f$ exist at every point of E. Suppose $Q \subset E$ is a closed rectangle with, side porolles to the coordinate axo having (a,b) and (a+h, b+k) as opposite vertes ($h \neq 0$, $k \neq 0$), suppose $\Delta(f,Q)=f(a+h,b+k)-f(a+h,b)-f(a,b+k)+f(a,b))$ then prove that there exists a point (x,y) in the interior of Q such that $\Delta(f,Q)=hk(D_{21}f)(x,y)$.

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10. a) State and prove vitali covering lemma.

(OR)

b) Prove that L^P spaces are complete for $1 \leq P < \infty$

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Code No. 1823

FACULTY OF SCIENCE

M.Sc. (Mathematics) II – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

FUNCTIONAL ANALYSIS

PAPER – III

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer the following questions in not more than **ONE** page each: (5x4=20)

1. Prove that on a finite dimensional vector space x, any norm ‖. ‖ is equivalent to any other norm ‖. ‖.
2. If M is any non-empty subset of a Hilbert space H, then prove that the span of M is dense in H if and only $M^{\perp} = \{0\}$.
3. If H is a separable Hilbert space, then prove that every orthonormal set in H is countable.
4. Let x be a normed linear space and $x_0 \neq 0$ be an element of x. Then prove that there exists a bounded linear functional $\bar{f}$ on x such that $\bar{f}(x_0) = \| x_0 \|$ and $\| \bar{f} \| = 1$
5. (i) Let T:H→H be a bounded linear operator on a complex Hilbert space H and <Tx, x> is real for all x∈H, then prove that the operator T is self-adjoint.
   (ii) Prove that any norm defined on Q vector space is sublinear functional.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) If X is a normed space and Y is a Banach space, then show that B(X,Y) is a Banach space.

   (OR)

   b) Let $\{x_1, x_2, \ldots, x_n\}$ be a linearly independent set of vectors in a normed space X of any dimension. Then prove that there is a number C>0 such that for every choice of scalars $\alpha_1, \alpha_2, \ldots \alpha_n$, we have $\|\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n\| \geqslant C(|\alpha_1| + |\alpha_1| + \cdots + |\alpha_1|)$.

7. a) Let X be an inner product space and M≠ϕ a convex subset which is complete(in the metric induced by the inner product). Then prove that for every given x∈X there exists a unique y∈M such that

   $$\sigma = \inf_{y \in M} \| x - \tilde{y} \| = \| x - y \|$$

   Also prove that if M=Y a subspace of x, then z=x-y is orthogonal to Y.

   (OR)

   b) Let (e_k) be an orhonormal sequence in an inner product space X. Then for every x∈X, prove that

   $$\sum_{k=1}^{\infty} |\langle x_1 e_k \rangle|^2 \leq \| x \|^2$$

8. a) State and prove Riesz representation theorem for sesquilinear form.

   (OR)

   b) Prove that the Hilbert-adjoint operator T^* of T exists, is unique and is a bounded linear operator with $\| T^* \| = \| T \|$.

9. a) State and prove Hahn-Banach theorem for normed spaces.

   (OR)

   b) Prove that a bounded linear operator T from a Banach space X onto a Banach space Y has the property that the image $T(B_o)$ of the open unit ball $B_o = B(0;1) \subset X$ contains an open ball about o∈Y.

10. a) (i) Prove that the product of two bounded self-adjoint linear operators A and B on a Hilbert space H is self-adjoint if and only if AB=BA.
   (ii) Let X and Y be normed spaces, Prove that the vector space X x Y is a normed space with respect to norm defined by $\|(x,y)\| = \max\{\|x\|_x, \|y\|_y\}$.

   (OR)

b) (i) Let H_1, H_2 be Hilbert spaces $S : H_1 \rightarrow H_2$ and $T : H_1 \rightarrow H_2$ bounded linear operators and $\propto$ any scalar. Then prove that (i) $(S+T)^*=S^*+T^*$ (ii) $(\propto T)^*=\overline{\propto}T^*$

(ii) Let X, Y be normed spaces and $S,T \in B(X,Y)$. Then prove that (i) $(S+T)^x=S^x+T^x$ (ii) $(\propto T)^x=\propto T^x$ for all scalars $\propto$.

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Code No. 1824

FACULTY OF SCIENCE

M.Sc. (Mathematics) II – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

PAPER – IV

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than **ONE** page each: (5x4=20)

1. Prove that there exist three linearly independent solutions of the third order equation $x'''+b_1(t)x''+b_2(t)x'+b_3(t)x=0$, $t\in I$, where b_1, b_2 and b_3 are functions defined and continuous on an interval I.
2. Solve the system of equation $x_1^1 = 5x_1 - 2x_2$
$$x_2^1 = 2x_1 + x_2$$
3. Show that $x(t)$ is a solution of initial value problem $x'=f(t,x)$, $x(t_0)=x_0$ on some interval I if only if $x(t)=x_0+\int_{to}^{t} f(s,x(s))ds$.
4. Define extremal solutions of the initial value problem $x'=f(t,x)$, $x(t_0)=x_0$ and hence find them for the Ivp $x'=3x^{2/3}$, $x_0=0$.
5. Compute the first three successive approximations for the solution of Ivp $x'=tx$, $x(0)=1$.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Show that there exist n linearly, independent solution on I of the n[th] order equation. $L(x(t))=x^{(n)}+b_1(t)x^{(n-1)}(t)+\ldots\ldots+b_n(t)x(t)=0$, $t\in I$. If $\phi_1,\phi_2,\ldots.\phi_n$ are linearly independent solution and ϕ is any other solution of above equation $L(x(t))=0$ existing on I, then show that there exist n constant $C_1,C_2,\ldots.C_n$ such that $\phi= C_1\phi_1,+C_2\phi_2+\ldots..+C_n\phi_n$; $t\in I$.

(OR)

b) State and prove Abel's formulae for n[th] order linear differential equation.

7. a) (i) Show that the necessary and sufficient condition for the system $x'=Ax$ to admit a non-zero periodic solution of period w in that $E-e^{Aw}$ in singular, where E is the identity matrix.
(ii) If f(t)is periodic with period w then prove that a solution $x(t)$ of $x'=Ax+f(t)$ in periodic of period w
if w as if $x(0)=x(w)$.

(OR)

b) If A(t) is an nxn matrix which is continuous on I and if a matrix ϕ satisfies $x'=A(t)X$, $t\in I$, then prove that det ϕ satisfies the equatition $\det\phi(t)=\det\phi(\tau)\exp\int_{\tau}^{t} tr\,A(s)ds$, for $\tau\in I$.

8. a) State and prove Picard's theorem for initial value problem $x'=f(t,x)$, $x(t_0)=x_0$.

(OR)

b) Define a fixed point of an operator T and state contraction principle. And also state and prove existence and uniqueness theorem of solution of initial value problem $x'=f(t,x)$, $x(t_o)=x_0$ using contraction principle.

9. a) State Ascoli's lemma. If f (t, x) is continuous and bounded on the infinite strip $S=\{(t,n) : t_0\le t\le t_0+h, |x|<\infty, h>0\}$, then prove that the initial value problem $x'=f(t,x)$, $x(t_0)=x_0$, has at least one solution existing on the interval $I=[t_0, t_0+h]$ using ascolilemna .

(OR)

b) Define upper and lower solution of the IVP $x'=f(t,x)$, $x(t_0)=x_0$. If u, $w\in C'\{(t_0,t_0+h),R\}$ are lower and upper solution of IVP $x'=f(t,x)$, $x(t_0)=x_0$ and if h for $x\ge y$, statistics the inequation $f(t,x)-(t,y)\le L(x-y)$, where L is a positive constant then prove that $\mho(t_0)\le w(t_0)$ implies $\mho(t)\le w(t)$, $t\in(t_0,t_0+h)$.

10. a) (i) Find the largest interval of existence of the solution of the IVP $n'=n^2$, $n(0)=2$ defined on $R=\{(t,x): |t|\le a, |x-2|\le b, a>0, b>0\}$.

P.T.O

(ii) Show that the function $f(t, x)=e^t \sin n$ satisfies lipschitz condition on the rectangle region $R=\{(t, x): |x| \leq 2\pi, |t| \leq 1\}$.

(OR)

b) If $f \in C[J \times R, R]$, $\mho_0$, w_0 are lower and upper solution of IVP $x'=f(t, x)$, $x(t_0)= x_0$ such that $\mho_0 \leq w_0$ on $I=[t_0, t_0+h]$ and further suppose that $f(t, x)-f(t,y) \geq -M(n-y)$ for $\mho_0 \leq y \leq n \leq w_0$ and $M \geq 0$, then prove that there existing monotone sequence $\{\mho_n\}$, $\{w_n\}$ such that $\mho_n \rightarrow \mho$ and $w_n \rightarrow w$ as $n \rightarrow \infty$ definition and monotonically as I where $\mho$ and w are minimal and maximal solution of above said IVP respectively.

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Code No. 1825

FACULTY OF SCIENCE

M.Sc. (Mathematics) II – SEMESTER REGULAR/BACKLOG EXAMINATIONS, MAY 2017

DISCRETE MATHEMATICS

PAPER – V

Time: 3 hours] [Max. Marks: 70

Note: Answer all the questions from Section – A and Section – B

Section – A

Answer any five of the following questions in not more than **ONE** page each: (5x4=20)

1. Define a poset and a chain. Give an example of a poset which is not a chain.
2. Obtain the sum of products canonical form of $[(x_1+x_2)(x_3x_4)^1]^1$.
3. Define a path, Simple path and an elementary path in a digraph with suitable example.
4. Prove that the number of vertices in one more than the number of edges in a tree.
5. Draw a binary tree where level order indices are {1, 2, 4, 5, 8, 10, 11, and 20}.

Section – B

Answer the following questions in not more than **FOUR** pages each: (5x10=50)

6. a) Let (L, ≤) be a lattice. Prove that for any a,b,c∈L the following inequality hold

   i) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$; $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

   ii) $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$

(OR)

b) i) Define the terms sub lattice, lattice homomorphism, complete lattice and complemented lattice.

   ii) State and prove De Morgan's laws in complimented distribution lattice.

7. a) (i) Prove that the direct product of any two Boolean algebra is also a Boolean algebra.

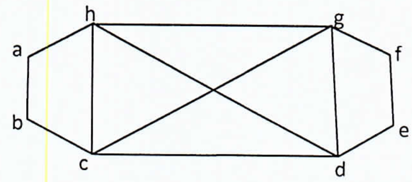
   (ii) State and prove the stone's representation theorem for finite Boolean algebra.

(OR)

b) Obtain a minimal Boolean expression by Karnaugh map representation for the following Boolean expression $f(x,y,z)=(x\oplus y)*(x^1\oplus z)*(y\oplus z)$.

8. a) i) Show that an indirected graph posses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree.

   ii) Find an Euler circuit in the following graph:



(OR)

b) i) Prove that if G is a connected plane graph then | v | - | E | + | R | = 2.

   ii) Prove that a complete graph K_n is planar if n≤4.

9. a) i) Prove that a tree with two or more vertices has at least two leaves.

   ii) Prove that a circuit and complement of any spanning tree must have at least one edge in common.

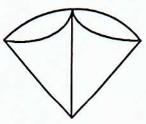
(OR)

b) i) Prove that connected graph always contains a spanning tree.

   ii) Define a cut set, prove that every circuit has an even number of edges common with every end set.

10. a) i) Prove that there is always a Hamiltanian path in a directed complete graph.

   ii) Is there an Eulerian path in the following graph?



(OR)

b) (i) Explain an algorithm to find minimum spanning tree of a given graph.

   (ii) Prove that a single non-directed graph G is a tree iff G is connected and containing no cycles.

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